MASS AND HEAT TRANSFER IN BUILDING MATERIALS

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A system of differential equations is proposed for mass and heat transfer in capillary-porous bodies. The principal thermophysical characteristics of mass and heat exchange in relation to building materials and structures are discussed.

Building materials are moist capillary-porous bodies, in which heat transfer is inseparably associated with mass transfer of moisture (vapor and liquid) and air. Therefore heat transfer calculations relating to building materials must take mass transfer into account. These calculations call for extensive use of machine computation and the application of generalized variable methods (similiarity theory). It is therefore necessary to have a sufficiently rigorous mathematical formulation of the problem and a clear understanding of the corresponding thermophysical characteristics.

Physical characteristics of the system. Consider a capillary-porous body with a lyophilic skeleton, i.e., one in which the walls of the capillaries and pores sorb gas, vapor and liquid; an osmotic-diffusion bond may then be formed between the substance sorbed and the skeleton.

A system of this kind differs substantially from the dispersed media models examined in classical filtration theory.

The substance bound by the capillary-porous skeleton may be in the form of a liquid, vapor, inert gas, solid, or supercooled liquid, depending on the conditions of mass and heat transfer. The freezing point of a liquid varies over a wide range, depending on the type of bond between substance and skeleton. There is therefore always a certain amount of supercooled liquid (water) in capillary-porous bodies at temperatures below 0°C.

It should also be noted that in most cases the pores and capillaries are not completely full of liquid or ice, but partly filled with a vapor-gas mixture. To simplify the analysis we shall assume the liquid to be water without any dissolved substance. The presence of dissolved substances modifies the mass transfer processes and causes a number of other effects.

The main parameter of a capillary-porous material is its moisture content u, or the sum of the relative concentrations of the i-th bound substance (i = 1, 2, 3, 4),

$$u = \sum_i u_i = \frac{\omega}{\gamma} = \frac{1}{\gamma}\sum_i \omega_i. \tag{1}$$

If shrinkage of the body is neglected, the volume concentration of the skeleton will equal the density of the absolutely dry material ($\gamma = \gamma_0$). If the skeleton walls are slightly hydrophilic, and changes in the density of the liquid of the monomolecular adsorbed layer are neglected, then the amount of moisture in the physico-mechanical bond (capillary moisture, ice, vapor, and air) may be determined from the relation

$$\omega_i' = \Pi_V b_i \rho_i = \gamma_0 u_i', \tag{2}$$

where b_i is the degree of filling of pores and capillaries, or the ratio of the volume of the i-th bound substance to the pore volume*;

$$\Pi_V = \int_{r_0}^{r_{\max}} f_V(r)\,dr;$$

is the volume porosity of the body; $f_V(r)$ is the differential pore distribution curve. The relation $\sum_i b_i = 1$ holds for b_i. It should be noted that (2) will not be valid for the moisture of the physico-chemical bond ($\omega_i \neq \omega_i'$; $u_i \neq u_i'$). The equality $\omega_i = \omega_i'$, $u_i = u_i'$, and hence (2), can be assumed only for typical capillary-porous materials with a small hygroscopic moisture content.

For the molar (hydrodynamic) motion of moisture (liquid, gas or vapor) through the pores that occurs in filtration processes the flow density of the i-th substance is

$$\mathbf{j}_{i\,\mathrm{mol}} = \Pi_S \rho_i b_i \mathbf{w}_i, \tag{3}$$

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*In filtration theory b_i is called the saturation.

where Π_S is the surface porosity of the body, or the ratio of the area of all the pores (holes) to the area of the cross section in a direction perpendicular to the vector $j_{i\,\mathrm{mol}}$.

For a polycapillary-porous body $\Pi_S = \int_{r_0}^{r_{\max}} f_S(r)\,dr$, where $f_S(r)$ is the differential distribution curve of the surface porisity of the body with respect to the radius of the pores. For homogeneous bodies the surface porosity Π_S is approximately equal to the volume porosity Π_V ($\Pi_V = \Pi_S = \Pi$).

Strictly speaking, the b_i in (3) is different from the b_i in (2). However, when $\Pi_V = \Pi_S = \Pi$, b_i will denote the degree of filling of the pores of the i-th bound substance. Thus we may write

$$\mathbf{j}_{i\,\mathrm{mol}} = \Pi \rho_i b_i \mathbf{w}_i. \tag{4}$$

Relation (4) may be used to determine the flow density of the capillary moisture in a monocapillary-porous body. In that case $w_i = w_{2\mathrm{cap}}$, where $w_{2\mathrm{cap}}$ is the linear velocity of the liquid in a monocapillary-porous material under the action of the capillary forces.

The following circumstance is also important. Under ordinary conditions, when the pressure of the moist air in the pores of the material is only slightly different from barometric, the mass of the air and vapor in the pores is negligibly small compared to that of the liquid or ice. Naturally, this presupposes that the material is in equilibrium with the surrounding moist air, i.e., its moisture content differs from zero. According to Posnov's calculations, under normal conditions for materials of maximum porosity (brick, wood, etc.) the mass of moist air in the pores of the material is about 10^{-5}% of the mass of liquid corresponding to the equilibrium moisture content. Therefore the total moisture content of the material u may be regarded as equal to the moisture content of liquid u_2 and ice u_3:

$$u = \sum_i u_i = u_2 + u_3. \tag{5}$$

We shall use this relation in calculating sources of bound matter due to phase transitions.

Differential equations of mass and heat transfer. From the law of conservation of mass of bound matter we obtain the differential equation of mass transfer

$$\frac{\partial (u_i \gamma)}{\partial \tau} = -\operatorname{div}(\mathbf{j}_{i\mathrm{dif}} + \mathbf{j}_{i\mathrm{mol}}) + I_i, \tag{6}$$

where the relation $\sum_i I_i = 0$ holds for I_i.

Neglecting shrinkage ($\gamma = \gamma_0 = \mathrm{const}$) and making use of (4), we get:

$$\gamma_0 \frac{\partial u_i}{\partial \tau} = -\operatorname{div}(\mathbf{j}_{i\mathrm{dif}} + \Pi \rho_i b_i \mathbf{w}_i) + I_i. \tag{7}$$

We obtain the differential equation of heat transfer from the equation of internal energy transfer

$$c\gamma_0 \frac{\partial T}{\partial \tau} = -\operatorname{div}\mathbf{j}_q + \sum_i h_i I_i - \sum_i (\mathbf{j}_{i\mathrm{dif}} + \Pi \rho_i b_i \mathbf{w}_i) c_i \nabla T, \tag{8}$$

where c is the reduced specific heat of the body* $\left(c = c_0 + \sum_i c_i u_i\right)$; h_i is the specific enthalpy of the i-th bound substance ($c_i = dh_i/dT$); j_q is the heat flux density due to Fourier-law thermal conduction ($j_q = -\lambda \nabla T$).

Equations (7) and (8) form the most general system of differential equations of mass and heat transfer in capillary-porous bodies, from which the equations for a number of special cases may be obtained: the filtration equation, the equations of moisture and heat transfer in monocapillary-porous materials, and so on. Vapor diffusion in capillary-porous bodies is determined by the moisture content and temperature gradients:

$$\mathbf{j}_{1\mathrm{dif}} = a_{m1}\gamma_0 \nabla u + a_{m_1}^T \gamma_0 \nabla T. \tag{9}$$

The flow of capillary moisture is proportional to the gradient of capillary potential, which in turn can be expressed in terms of the moisture content and temperature gradients, i.e., the motion of capillary moisture can be considered as capillary diffusion.

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*For a solid the specific heat at constant volume c_V is usually assumed to be equal to the specific heat at constant pressure, c_p, i.e., $c_p = c_V = c$.

Consequently, in a porous material with a polycapillary structure, diffusion of osmotically bound liquid and capillary diffusion may be represented by a single vector – the flow of diffusion and capillary moisture:

$$\mathbf{j}_{2\mathrm{dif}} = a_{m2}\,\gamma_0 \nabla u + a_{m2}^{T}\,\gamma_0 \nabla T. \tag{10}$$

The sources of bound matter I_i ($i = 1, 2, 3, 4$) may be expressed as follows. When there are no chemical transitions, $I_4 = 0$ (absolutely dry air is considered an inert gas). When liquid turns into ice, the ice source I_3 may be determined in terms of the ice ratio $\varepsilon_{\mathrm{ice}}$, the ratio of mass of ice to mass of liquid and ice $[\varepsilon_{\mathrm{ice}} = u_3/u = u_3/(u_2 + u_3)]$:

$$I_3 = -I_2 = \frac{\varepsilon_{\mathrm{ice}}}{1-\varepsilon_{\mathrm{ice}}}\,\gamma_0 \frac{\partial u_2}{\partial \tau} = \varepsilon_{\mathrm{ice}}\gamma_0 \frac{\partial u}{\partial \tau}. \tag{11}$$

Here it is supposed that the mass of vapor in the pores is negligibly small compared with the mass of liquid and ice $[u_1 \ll (u_2 + u_3)]$. Source I_2 ($I_2 = -I_1$) may be determined from (6) when $i = 1$, by putting $\partial u_1/\partial\tau = 0$ in accordance with the above reasoning, i.e.,

$$I_2 = -I_1 = -\operatorname{div}(\mathbf{j}_{1\mathrm{dif}} + \mathbf{j}_{1\mathrm{mol}}). \tag{12}$$

If transfer of moisture proceeds by means of diffusion (molecular and capillary), then the system of differential equations for mass and heat transfer will have the form

$$\gamma_0 \frac{\partial u_i}{\partial \tau} = \operatorname{div}(a_{mi}\,\gamma_0 \nabla u + a_{mi}^{T}\gamma_0 \nabla T) + I_i, \tag{13}$$

$$c\gamma_0 \frac{\partial T}{\partial \tau} = \operatorname{div}(\lambda \nabla T) + \sum_i r_i I_i - \sum_i \mathbf{j}_{i\mathrm{dif}} c_i \nabla T. \tag{14}$$

Summing over all i in (13), we obtain

$$\frac{\partial u}{\partial \tau} = \sum_i \operatorname{div}(a_{mi} \nabla u + a_{mi}^{T} \nabla T). \tag{15}$$

The value of heat transfer due to diffusion of enthalpy $\sum_i \mathbf{j}_{i\mathrm{dif}} c_i \nabla T$ may be neglected. Then, using the expressions for sources I_i, we obtain the system of differential equations of mass and heat transfer in the final form

$$\frac{\partial u}{\partial \tau} = \operatorname{div}(k_{11} \nabla u) + \operatorname{div}(k_{12} \nabla T), \tag{16}$$

$$c\,\frac{\partial T}{\partial \tau} = \operatorname{div}(k_{22}' \nabla T) + \operatorname{div}(k_{21}' \nabla u). \tag{17}$$

In the zonal calculation system, coefficients k_{11}, k_{12}, k_{22}', k_{21}' may be assumed constant in each separate interval Δu and ΔT. Then the system of Eqs. (16), and (17) may be written:

$$\frac{\partial u}{\partial \tau} = k_{11} \nabla^2 u + k_{12} \nabla^2 T, \tag{16'}$$

$$\frac{\partial T}{\partial \tau} = k_{22} \nabla^2 T + k_{21} \nabla^2 u. \tag{17'}$$

Here $k_{22} = 1/c(k_{22}')$, $k_{21} = 1/c(k_{21}')$ and the coefficients k_{12} and k_{21} are not equal. The coefficients k_{ij} ($i, j = 1, 2$) are determined by the state of the moisture in the material.

Then for the system $i = 1, 2$

$$k_{11} = a_m = (a_{m1} + a_{m2}), \quad k_{12} = a_m^{T} = a_m \delta,$$

$$k_{22} = a + a_{m1}\frac{r_{12}}{c}, \quad k_{21} = a_{m1}^{T}\frac{r_{12}}{c} = a_{m1}\delta_1 \frac{r_{12}}{c};$$

for the system $i = 2, 3$

$$k_{11} = a_{m2}(1-\varepsilon_{\text{ice}}), \quad k_{12} = (1-\varepsilon_{\text{ice}})a^T_{m2},$$

$$k_{22} = a + (1-\varepsilon_{\text{ice}})a^T_{m2}\frac{r_{23}}{c}, \quad k_{21} = (1-\varepsilon_{\text{ice}})a_{m2}\frac{r_{23}}{c};$$

for the system $i = 1, 3$

$$k_{11} = a_{m1}, \quad k_{12} = a^T_{m1}, \quad k_{22} = a + a^T_{m1}\frac{r_{13}}{c}, \quad k_{21} = a_{m1}\frac{r_{13}}{c}.$$

Thermophysical characteristics. The system of Eqs. (16) and (17) describes mass and heat transfer by diffusion in a capillary-porous body. The transfer coefficients entering into the system as thermophysical characteristics are as follows: moisture diffusion coefficient (vapor and liquid) a_m, or the sum of the diffusion coefficients of moisture in vapor a_{m1} and liquid a_{m2} from ($a_m = a_{m1} + a_{m2}$), the thermal diffusivity or coefficient of diffusion of heat (energy) a, the coefficient of thermal diffusion of moisture (vapor and liquid) a^T_m, or the sum of the coefficients of thermal diffusion of moisture in vapor a^T_{m1} and liquid a^T_{m2} from ($a^T_m = a^T_{m1} + a^T_{m2}$). In addition, the following thermodynamic properties appear: the specific heat of the material c and the heat of phase transition r.

Consequently, to describe mass and heat transfer in capillary-porous materials, the following thermophysical properties must be known: a_{m1}, a_{m2}, a^T_{m1}, a^T_{m2}, a, c and r, i.e., seven given values for each material. Instead of these seven characteristics, the following seven may be taken: a_{m1}, a_{m2}, δ_1, δ_2, λ, c and r, where $\lambda = ac\gamma$, δ_1 and δ_2 are the thermogradient coefficients, or the ratio of the coefficients of thermal diffusion to the coefficients of diffusion of vapor and liquid moisture ($\delta_1 = a^T_{m1}/a_{m1}$; $\delta_2 = a^T_{m2}/a_{m2}$). The relation between the diffusion coefficients and thermogradient coefficients is

$$\delta = \frac{a^T_m}{a_m} = \frac{a^T_{m1} + a^T_{m2}}{a_{m1} + a_{m2}} = \frac{a_{m1}\delta_1 + a_{m2}\delta_2}{a_{m1} + a_{m2}}. \tag{18}$$

The thermophysical properties a_{m1}, a_{m2}, δ_1, δ_2, λ, c and r are determined by experiment. The coefficients of moisture diffusion a_m are usually determined by drying samples of moist material in an atmosphere of moist air, and the coefficients of liquid diffusion a_{m2} by immersing samples of moist material in water. The experimental determination of the thermogradient coefficient δ is not diffucult; it is numerically equal to the fall in moisture content for a temperature drop of one degree with no moisture transfer (in the steady state), i.e., $\delta = (\Delta u/\Delta T)_{j=0}$.

The determination of the coefficients δ_1 and δ_2 presents some difficulty. In the hygroscopic region, however, the equality $\delta_1 = \delta_2 = \delta$ may be considered valid; this is conditioned by thermodynamic and molecular equilibrium between liquid and vapor inside the moist material.

In drying processes, which involve typical unsteady mass and heat transfer, the coefficient of thermal diffusion of vapor a^T_{m1} is replaced by the phase transition coefficient ε, which enters into the expression for the liquid drain (or source)

$$I_2 = -I_1 = \varepsilon\gamma_0\frac{\partial u}{\partial \tau}. \tag{19}$$

This relation is derived from the balance equation of conservation of mass of liquid in the moist material. The coefficient ε is equal to the ratio of change of liquid due to evaporation or condensation du_l to the overall change of moisture content of the material in an infinitely small volume du, i.e., $\varepsilon = du_l/du$.

The coefficient ε varies from zero to unity ($0 < \varepsilon < 1$). Relation (19) is valid only for unsteady mass transfer. For steady mass transfer, (19) gives an indeterminancy ($\partial u/\partial \tau = 0$, $\varepsilon = \infty$), which when expanded leads to the usual expression for a moisture source.

Using the differential equations

$$\frac{\partial u_1}{\partial \tau} = -\operatorname{div}\mathbf{j}_1 + I_1, \quad \frac{\partial u}{\partial \tau} = -\operatorname{div}\mathbf{j}_1 - \operatorname{div}\mathbf{j}_2 \tag{20}$$

we obtain

$$(I_2)_{u=\text{const}} = -(I_1)_{u=\text{const}} = \varepsilon\gamma_0\frac{\partial u}{\partial \tau} = \operatorname{div}\mathbf{j}_2 = -\operatorname{div}\mathbf{j}_1. \tag{21}$$

For unsteady mass transfer, putting $\partial u_1/\partial\tau = 0$, we have from (19) and (20)

$$\varepsilon = \operatorname{div} \mathbf{j}_1/(\operatorname{div} \mathbf{j}_1 + \operatorname{div} \mathbf{j}_2). \tag{22}$$

For the one-dimensional problem ($\operatorname{div} \equiv \partial/\partial x$) the phase transition coefficient is numerically equal to the ratio of vapor flux to the total flux of vapor and liquid

$$\varepsilon = |j_1|/(|j_1| + |j_2|), \tag{23}$$

in which it is assumed that the coefficient ε is a constant.

If (19) is substituted for $I_2 = -I_1$ in the system of differential Eqs. (13) and (14), Eqs. (16) and (17) are obtained, in which the coefficients k_{ij} $(i, j = 1, 2)$ are determined by*:

$$k_{11} = a_m = \frac{a_{m2}}{1-\varepsilon}, \quad k_{12} = \frac{a_{m2}\,\delta_2}{1-\varepsilon} = a_m\,\delta, \tag{24}$$

$$k_{22} = a + \varepsilon\frac{r_{21}}{c}a_m\,\delta, \quad k_{21} = \varepsilon\frac{r_{21}}{c}\,\delta\,a_m. \tag{25}$$

Thus, mass and heat transfer in a capillary-porous body will be described by the system of differential Eqs. (16) and (17) along with the thermophysical characteristics a_m, a, δ, ε, c and r, i.e., six given values for the given moist material.

Introducing the coefficient ε thus reduces the number of thermophysical characteristics from seven to six. In the hygroscopic region this is equivalent to assuming the equality $\delta_1 = \delta_2 = \delta$. This system of characteristics is less general than the first, however, because it is valid only for unsteady mass and heat transfer.

Boundary conditions. Buildings are essentially systems of contiguous moist capillary-porous bodies. We may assume approximately that at the surfaces of contact between the moist bodies equality of the moisture j_k and heat q_k fluxes obtains, together with equality of moisture and heat transfer potentials (θ, T), i.e.,

$$\mathbf{j}_{k+1} = \mathbf{j}_k, \quad \mathbf{q}_k = \mathbf{q}_{k+1}, \tag{26}$$

$$\theta_{k+1} = \theta_k, \quad T_k = T_{k+1}, \tag{27}$$

where θ is the moisture transfer potential, measured in degrees of mass transfer (DM), and the subscript k denotes the k-th layer of moist material, in contact with the (k + 1)-th layer.

There is a relation between the moisture transfer potential θ and the moisture content, analogous to that between enthalpy (heat content) and temperature, i.e.,

$$c_m = \left(\frac{\partial u}{\partial\theta}\right)_T, \tag{28}$$

where c_m is the specific isothermal mass capacity (moisture capacity) of the body.

The thermodynamic basis of the moisture transfer potential has been given in [4], which also gives tables of specific mass capacity for a large number of moist materials. Methods have now been developed for experimentally determining the moisture transfer potential for various moist materials.

Mass and heat transfer take place at the boundary between moist air and the surface of the material. The interaction of the surface of the moist material and the air is usually described by boundary conditions of the third kind. Recent work by the author has shown that boundary conditions of the third kind are incorrect for unsteady mass and heat transfer, since the mass and heat transfer coefficients are functions of time. Boundary conditions of the third kind are valid only for steady mass and heat transfer.

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*Expression (23) can be used as a basis for deriving the moisture source, as in [1]. It follows from (23) that $|j_1| = (\varepsilon/1 - \varepsilon)|j_2|$. Let us denote the unit vectors along j_1 and j_2 by 1_{n1} and 1_{n2}, respectively. We may then write $j_1 = 1_{n1}|j_1|$; $j_2 = 1_{n2}|j_2|$. From the differential equation of moisture transfer (20) we obtain $\gamma_0(\partial u/\partial\tau) = -\operatorname{div} j_1 - \operatorname{div} j_2 = -\operatorname{div}[1_{n1}|j_1|] - \operatorname{div}[1_{n2}|j_2|] = -\operatorname{div}(\varepsilon/1 - \varepsilon)1_{n1}|j_2| - \operatorname{div} j_2$. If we put $1_{n1} = 1_{n2}$ (one-dimensional problem or equality of thermogradient coefficients $\delta_1 = \delta_2$) and assume ε independent of the coordinates (zonal system of calculation), we get $\gamma_0(\partial u/\partial\tau) = -(\varepsilon/1-\varepsilon) \operatorname{div} j_2 - \operatorname{div} j_2 = -\operatorname{div} j_2 + \varepsilon\gamma_0(\partial u/\partial\tau)$. Consequently, the moisture source $I_2 = \varepsilon\gamma_0(\partial u/\partial\tau)$.

The outside walls of buildings involve problems of unsteady mass and heat transfer, and it is therefore necessary to use boundary conditions of the second kind for a more accurate representation of the actual transfer process between the surface of the material and the surrounding medium.

$$-a_m(\nabla u)_s - a_m^T(\nabla T)_s + \mathbf{j}_s(\tau) = 0, \tag{29}$$

$$-\lambda(\nabla T)_s + \mathbf{q}_s(\tau) - r_{12}(a_{m2}\nabla u + a_{m2}^T\nabla T)_s = 0, \tag{30}$$

where the subscript s denotes the surface of the material, $q_s(\tau)$ is the variable heat flux at the surface, or the sum of the radiant and convective heat fluxes, and $j_s(\tau)$ is the variable moisture flux at the surface.

Therefore, to describe the interrelated mass and heat transfer between moist building materials and the surrounding medium, the following thermophysical characteristics must be known: transfer coefficients a, λ, a_m, a_{m2}, a_m^T, a_{m1}^T, the specific heat and mass capacity c and c_m, and the specific heat of phase transition. When ice is present, the ice ratio ε_{ice} must also be known. Interaction with the surrounding medium is described by the dimensionless Kirpichev numbers

$$\mathrm{Ki}_q(\tau) = \frac{q_s(\tau)\, l}{\lambda T_c} \text{ and } \mathrm{Ki}_m(\tau) = \frac{j_s(\tau)\, l}{a_m \gamma_0 u_e}, \tag{31}$$

where l is a characteristic dimension; T_c is a characteristic temperature: and u_e is the equilibrium moisture content. In most cases $\mathrm{Ki}_q(\tau)$ and $\mathrm{Ki}_m(\tau)$ are periodic functions of time and are determined by the parameters of the surrounding medium (temperature, humidity, and velocity of the air, and the intensity of radiative heat transfer).

Finally, it should be noted that in certain special cases the system of differential Eqs. (16), (17) may be simplified. For example, in the region of the moisture state the system may be written as:

$$\frac{\partial u}{\partial \tau} = a_m \nabla^2 u + a_m^T \nabla^2 T, \tag{32}$$

$$\frac{\partial T}{\partial \tau} = \left(a + \frac{r_{12}}{c} a_{m1}^T\right)\nabla^2 T. \tag{33}$$

The last equation is the usual Fourier heat conduction equation with the thermal diffusivity $a_{eq} = a + (r_{12}/c)a_{m1}^T$. When filtration mass transfer is present, determined by the relation $j_f = -k_f\nabla p$, where k_f is the coefficient of filtration moisture transfer, the system of differential equations of heat and mass transfer will have the form

$$\frac{\partial u}{\partial \tau} = \mathrm{div}\,(a_m \nabla u + a_m^T \nabla T + a_f \nabla p), \tag{34}$$

$$c_f \gamma_0 \frac{\partial p}{\partial \tau} = \mathrm{div}\,(k_f \nabla p) + I_1, \tag{35}$$

$$c\gamma_0 \frac{\partial T}{\partial \tau} = \mathrm{div}\,(\lambda \nabla T) + r_{12} I_2 + \sum_{i=1}^{2} k_{fi} c_i \nabla T \nabla p, \tag{36}$$

where a_f is the convective diffusion coefficient ($a_f = k_f/c_f\gamma_0$); c_f is the capacity coefficient of moist air in the porous material, determined by the relation $d(u_1 + u_4) = c_f dp$. Here we neglect enthalpy transfer due to diffusion of moisture.

This system of Eqs. (34)-(36) may also be simplified and reduced to a system of differential equations of the following type*:

$$\frac{\partial \vartheta_i}{\partial \tau} = \sum_j \mathrm{div}\,(k_{ij} \nabla \vartheta_i);\ i,\ j = 1,\ 2,\ 3, \tag{37}$$

where $\vartheta_1 = u$, $\vartheta_2 = T$, $\vartheta_3 = p$.

The coefficients k_{ij} are functions of u and T, and system of differential Eqs. (37) is solved for given thermophysical characteristics using computers. Therefore the first task of building heat physics is to develop experimental methods for determining the thermophysical characteristics of moist building materials as a function of the moisture content and temperature.

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*A detailed derivation of this system is given in monograph [2], together with a number of solutions for various boundary conditions.

NOTATION

ω_i – volume concentration of i-th bound substance; γ – volume concentration of skeleton of material, or the ratio of mass of skeleton to volume of material; ρ_i – density of i-th state of moisture; u – total moisture content of material; w_i – average linear velocity of molar (filtration) motion of i-th substance; τ – time; T – temperature; j_{idif} – diffusion current density of i-th bound substance; I_i – source strength of i-th bound substance due to phase transitions; λ – thermal conductivity of moist material; c – specific heat of material; r – specific heat of phase transition; a_m – diffusion coefficient of moisture (vapor and liquid) in moist material; a_{m1} – diffusivity of water vapor (diffusivity of vapor in moist material); a_{m2} – diffusivity of liquid water (diffusivity of liquid in moist material); a_{m1} and a_{m2} – respectively, coefficients of thermal diffusion of vapor and liquid in moist material; p – total pressure inside moist material. Subscripts: 1 – water vapor; 2 – liquid; 3 – moisture in third state (ice); 4 – inert gas (dry air); 0 – skeleton.

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